

Reg. No. :

K23U 0514

Name :

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/
Improvement) Examination, April 2023
(2019 and 2020 Admissions)

CORE COURSE IN MATHEMATICS

6B11 MAT : Complex Analysis

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark :

1. Check whether $u = e^x \sin 2y$ is harmonic or not.

2. Evaluate $\int_{-\pi i}^{\pi i} \cos z \, dz$.

3. State Cauchy's integral theorem.

4. Discuss the convergence of $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$.

5. Write the Maclaurin series for $\sin z$.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks :

6. Find real part and imaginary part of $f(z) = \frac{1}{1-z}$ at $1-i$.

7. Check whether $f(z) = \cos x \cosh y - i \sin x \sinh y$ is analytic.

8. Define an entire function and write example of an entire function.

9. Evaluate $\int_C \operatorname{Re} z \, dz$, where C is the shortest path from $1+i$ to $3+3i$.

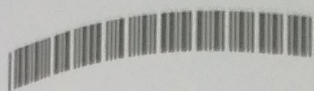
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10. Determine $\int_C \frac{1}{2z-1} dz$, where C is the unit circle in the counter clock wise direction.
11. Prove that if a series $z_1 + z_2 + \dots$ converges, then $\lim_{n \rightarrow \infty} z_n = 0$.
12. State root test for the convergence of a series.
13. Check the convergence of $\sum_{n=0}^{\infty} \frac{1^n}{n^2 - i}$.
14. State Laurent's theorem.
15. Evaluate $\oint_C \frac{1}{(z-1)(z-3)} dz$, $C: |z| = \frac{3}{2}$, in the counter clock wise direction.
16. Define zeros and singularities of a function $f(z)$ and write example for each.

PART - C

Answer **any four** questions. **Each** question carries **four** marks :

17. Show that $f(z) = \bar{z}$ is nowhere differentiable.
18. Prove that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.
19. State and prove Cauchy's integral formula.
20. State and prove Morera's theorem.
21. Define radius of convergence of a power series also find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$.
22. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0.
23. Find the residues at singular points of $\frac{\sin z}{z^3 - z}$.



Answer any two questions. Each question carries six marks :

24. a) Find the value of z when $\ln z = 4 - 3i$.
b) Express i^i in the form of $a + ib$.
c) Write $e^{2+3\pi i}$ in the form of $u + iv$ also find $|e^{2+3\pi i}|$.
25. Evaluate using Cauchy's integral formula.
a) $\oint_C \frac{e^z}{z^n} dz$, where C is the unit circle in the counter clock wise direction.
b) $\oint_C \frac{z+2}{z-2} dz$, $C: |z-1|=2$, in the counter clock wise direction.
26. a) Find Maclaurin series for $f(z) = \sin(2z^2)$.
b) Find Taylor series for $f(z) = \frac{1}{(z+i)^2}$ with center $z_0 = i$, also find radius of convergence.
27. a) State and prove Cauchy's residue theorem.
b) Evaluate $\oint_C \frac{dz}{z^3(z-1)}$, $C: |z|=2$, in the counter clock wise direction.

